Imitation Monte Carlo methods for Boltzmann equation problems, parallelization of the algorithms with splitting.

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The talk examines a system of pairwise interaction particles, which models a rarefied gas in accordance with the nonlinear Boltzmann equation, the master equations of Markov evolution of this system and corresponding numerical Monte Carlo methods. Selection of some optimal method for simulation of rarefied gas dynamics depends on the spatial size of the gas flow domain. For problems with the Knudsen number Kn of order unity quite adequate and competitive are "imitation", or "continuous time", Monte Carlo methods ([1]). However if $Kn \leq 0.1$ (the large sizes) , excessive punctuality, namely, the need to see all the pairs of particles in the latter, leads to a significant increase in computational cost(complexity). We are interested in to construct the optimal methods for Boltzmann equation problems with large enough spatial sizes of the flow. Speaking of the optimal, we mean that we are talking about algorithms for parallel computation to be implemented on high-performance multi-processor computers. The characteristic property of large systems is the weak dependence of sub-parts of each other at a sufficiently small time intervals. This property is taken into account in the approximate methods using various splittings of operator of corresponding master equations. In the talk, we develop the approximate method based on the splitting of the operator of master equations system "over groups of particles" ([2]). The essence of the method is that the system of particles is divided into spatial subparts which are modeled independently for small intervals of time, using the precise"imitation" method. The type of splitting used is different from other well-known type "over collisions and displacements", which is an attribute of the known Direct simulation Monte Carlo methods. The second attribute of the last ones is the grid of the "interaction cells", which is completely absent in the imitation methods. The main of talk is parallelization of the imitation algorithms with splitting using the MPI library. New constructed algorithms are applied to solve the problems: on plane Poiseuille flow in the field of external forces and on propagation of the temperature discontinuity. In particular, on the basis of numerical solutions, comparative estimates of the computational cost are given for all algorithms under consideration.

References

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[2] *A.I. Khisamutdinov, N.N. Velker* . On Reduction of Computational Cost of Imitation Monte Carlo Algorithms for Modeling Rarefied Gas Flows// Mathematical Models and Computer Simulations, 2012, Vol. 4, No. 2, pp. 187-202.